Analysis of the addition of a third algal group to the Bays Eutrophication Model (BEM) kinetics

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SECTION 1

INTRODUCTION

1.1 HISTORY OF THE BAYS EUTROPHICATION MODEL (BEM)

In 1991, the Massachusetts Water Resources Authority (MWRA) funded the development of a coupled hydrodynamic/water quality model of Massachusetts and Cape Cod Bays as part of their Harbor and Outfall Monitoring Program (HOM). This model, the Bays Eutrophication Model (BEM), was developed to assess the potential impact of relocating the discharge of primary treated effluent of the Nut and Deer Island wastewater treatment plants from Boston Harbor into Massachusetts Bay. The initial calibration of the model was completed in 1995 and was conducted for the periods of October 1989 to May 1991 and the calendar year 1992. The Model Evaluation Group (MEG), assembled to provide a peer review for BEM, recommended, among other things, that additional validation of the model be conducted for the years 1993 and 1994. Events occurred in 1993 (a fall diatom bloom) and 1994 (a low dissolved oxygen event) that were considered by the MEG to be good tests for the model's predictive capability.

In March 2000, HydroQual released its report "Bays Eutrophication Model (BEM): Modeling Analysis for the Period of 1992-1994" to present the results of the additional model validation for MEG review. In June of 2000, the MEG met to present their conclusions and recommendations concerning the modeling study to the Outfall Monitoring Science Advisory Panel (OMSAP). While the MEG was pleased that the additional modeling that they requested had been completed it was believed that further analysis should be conducted. Specific events that occurred in 1993 and 1994 were not reproduced accurately enough to satisfy the MEG. Recommendations for additional analysis included: the exploration of the addition of a third algal group to the model kinetics; running the water quality model on the same spatial grid as the circulation model; sensitivity analysis of the upstream boundary conditions; as well as additional documentation for the model. This report addresses the addition of a third algal group.

1.2 RATIONALE FOR THE ADDITION OF A THIRD ALGAL GROUP

Traditionally, eutrophication models have included two functional phytoplankton groups to represent the 50 to 100 algal species that may be present in a particular body of water at any particular time. The two functional groups have been divided into cold water species, mainly diatoms, and warm water species, representing greens, blue-greens, flagellates, etc. These two groups differ in their

temperature, nutrient and light requirements and can potentially differ in other characteristics such as maximum growth rates, settling rates, and/or carbon to chlorophyll ratios. Depending on the environmental conditions, one group may have a competitive advantage over the other and may dominate the algal population during portions of the year. Historically, this type of model is able to reproduce a spring bloom, and the summer algal biomass, but often misses fall blooms when they occur. BEM falls into this category.

The rationale behind adding a third algal group is to develop the ability to reproduce the fall bloom. The difficulty of adding the third algal group is the specification of the proper temperature, light and nutrient requirements that will allow this third algal group to dominate during the fall and only during the fall, and also to be abundant only during years when the fall bloom is observed. This is especially difficult when the biology of why species-specific fall blooms occur is not fully understood. Thus, a model may be able to reproduce a fall bloom during a particular year for the wrong reasons, thereby limiting the model's predictive capability.

1.3 REPORT OUTLINE

The report documents the exploration of the addition of a third algal group to the BEM kinetics as requested by the MEG. Section 2 presents the model's algal kinetics as a background for the results that follow. Section 3 presents the results of the new model's kinetics and a comparison to the original calibration. Model sensitivities are also presented in Section 3. In Section 4, the conclusions from this analysis are presented.

SECTION 2

KINETICS

2.1 PHYTOPLANKTON KINETICS

The new version of the water quality model considers three functional phytoplankton groups: winter, summer, and fall. These distinctions are made to recognize some of the physiological differences between the phytoplankton species that dominate in each of these seasons in terms of optimal temperature and light conditions and nutrient requirements. The winter functional group is characterized as favoring low temperature and light conditions and as having a high requirement for silica, as well as nitrogen and phosphorus, as a nutrient source. The summer group represents a mixed population of phytoplankton, including greens, blue-greens, dinoflagellates and some diatoms. This group favors higher temperature and light conditions and has lower silica requirements than does the winter group. The fall group is a second diatom group with a mid-range temperature optimum and a lower nitrogen requirement.

The kinetic framework used for each of the functional algal groups is largely the same. Differences between the groups are expressed by the choice of model coefficients. Conventionally, eutrophication based water quality models consider phytoplankton growth to be a function of temperature, light and available nutrients, as shown in Equation (2-1):

$$G_{Pmax} = G_{Pmax} (T_{opt}) \cdot G_T (T) \cdot G_I (I) \cdot G_N (N)$$

temperature light nutrients (2-1)

where

G_{Pmax}	=	gross phytoplankton growth rate,
$G_{Pmax}(T_{opt})$	=	nutrient saturated growth rate at the optimal temperature,
$G_{T}(T)$	=	the reduction factor caused by temperature,
$G_{I}(I)$	=	the reduction factor caused by light attenuation,
$G_N(N)$	=	the reduction factor caused by nutrient limitation.
G _I (I) G _N (N)	= =	the reduction factor caused by light attenuation, the reduction factor caused by nutrient limitation.

The algal growth model used in this study draws directly from Laws and Chalup (1990) and an earlier modeling framework developed by Shuter (1979). The following paragraphs provide an overview of the Laws/Chalup model. In the Laws/Chalup model, the carbon in the phytoplankton cell is

considered to be found in one of four compartments: structural carbon (S), reservoir or storage carbon (R), carbon associated with the light reactions (photochemical reactions) of photosynthesis (L), or carbon associated with the dark reactions (carbohydrate production and protein and lipid synthesis) of photosynthesis (D). Hence, total cell carbon, C = S + R + L + D. Chlorophyll is assumed to exist only in the L portion of the cell. Nutrients (nitrogen, phosphorus, and silica) are found in the S, L, and D portions of the cell and are assumed to be found in the same ratios in each of these pools. R is assumed to consist entirely of C storage products (carbohydrates and lipids) and hence contains no nutrients. The fraction of C allocated to structural purposes (S/C) is assumed to be constant and independent of growth conditions.

The steady-state gross photosynthetic rate per cell (ρ) is described by

$$\rho = G_{\mathbf{n}\mathbf{l}} \mathbf{L} \mathbf{I} = G_{\mathbf{n}\mathbf{d}} \mathbf{D} \tag{2-2}$$

where I is the incident irradiance; G_{prd} is the gross photosynthetic rate per unit D per day and is a constant; and G_{prl} is the gross rate of photosynthesis per unit L per unit light intensity and is a function of environmental conditions. Respiration losses are assumed to be described by

$$k_{PR} C = k_{RB} C + k_{RG} G_{prd} D$$
(2-3)

where k_{RB} is the basal respiration rate per day, i.e., the rate required to maintain the cell in the absence of growth, and k_{RG} is the growth-rate-dependent respiration coefficient (Laws and Bannister, 1980). The substrate for respiration is assumed to come from the R pool.

From the foregoing assumptions, it follows that the rate of nutrient assimilation $f_{\rm N}$ is constrained by

$$\frac{d}{dt} (S+L+D) = W_{0x} f_{N}$$
(2-4)

where W_{Cx} is the ratio of C to nutrient x (either nitrogen, phosphorus or silica). It also follows that

$$\frac{dC}{dt} = G_{prd} D - k_{RB} C - k_{RG} G_{prd} D$$
(2-5)

Under conditions of balanced growth it must be true that for any component X of the cell,

$$\mu = \frac{1}{X} \frac{\mathrm{d}X}{\mathrm{d}t} \tag{2-6}$$

where μ is the growth rate in units of inverse time. Combining Equations (2-5) and (2-6) yields

$$\mu C = G_{prd} D - k_{RB} C - k_{RG} G_{prd} D$$
(2-7)

Laws and Chalup then go on to define the assumptions and conditions under which a nutrient saturated version of Equation (2-7) can be developed. The nutrient saturated growth rate, μ_{Pmax} , is of the form

$$\mu_{\rm Pm\,ax} = \frac{G_{\rm prd} (1 - k_{\rm RG}) (1 - S/C)I}{I + G_{\rm prd} / G_{\rm prls}} - k_{\rm RB}$$
(2-8)

where G_{prls} is the nutrient-saturated value of G_{prl} . Laws and Chalup then account for the relationship between light and G_{prls} by use of Equation (2-9).

$$G_{prk} = \frac{G_{prk}}{1+I/I_s}$$
(2-9)

where G_{prlo} is the value of G_{prls} when I = 0, and I_s is the value of I when $G_{prls} = 0.5G_{prlo}$. Laws and Chalup also continue the development of their model and, via algabraic equations and simplifying assumptions, reduce the foregoing equation to ones based on the total carbon pool. It is from this point that the remainder of this section is developed.

In the natural environment, the light intensity or incident irradiance, I, to which the phytoplankton are exposed is not uniformly at the optimum value. At the surface and near-surface of the air-water interface, photosynthesis occurs at or near maximum rates due to high light intensities, while at depths below the euphotic zone, light is not available for photosynthesis due to attenuation by background and algal related turbidity. During the day the light intensity at the air-water interface varies as a function of the angle of the sun relative to the horizon. Therefore, the BEM framework was designed to account for both of these factors. To account for the effect of variations of available light

as a function of depth, the light intensity, I(z), at any depth, z, is related to the incident surface intensity, I_{surf} , via the extinction coefficient, k_e , through the formula $I(z) = I_{surf} \exp(-k_e z)$. The average light intensity to which the phytoplankton are exposed within a water column slice of thickness H may be obtained from the following integral:

$$I_{av} = \frac{1}{H} \int_0^H I(z) dz$$
(2-10)

where:

I(z)	=	$I_{surf} e^{-k_e z}$,
e	=	2.718,
Н	=	thickness of water column slice (m),
k _e	=	the total extinction coefficient, computed from the sum of the base (non-algal
		related) light attenuation, $k_{\mbox{\tiny ebase}}$ and the self-shading attenuation due to the
		ambient phytoplankton population, $k_c P_{chl-a}$, (m ⁻¹),
k _{ebase}	=	the base extinction coefficient due to background conditions created by natural
		turbidity and exogenous suspended solids (m ⁻¹),
k _c	=	the algal related extinction coefficient per unit chlorophyll (m ² /mg chl-a),
P_{chl-a}	=	the ambient phytoplankton population as chlorophyll (mg chl-a/L), where P_{chl-a}
		$= a_{\text{ChlC}} P_{c},$
P _c	=	the ambient phytoplankton population as carbon (mg C/L),
a_{ChlC}	=	the ratio of algal chlorophyll to algal carbon (mg chl-a/mg C),

The result of this integral is:

$$I_{av} = \frac{I_{suf}}{k_e H} \left(1 - e^{-k_e H} \right)$$
(2-11)

The value of the surface light intensity, I_{surf} may be evaluated at any time within the day, t, using the following formula:

$$I_{surf}(t) = \frac{I_{tot}}{0.635f} sin\left[\frac{\pi(t_d - t_{surrise})}{f}\right]$$
(2-12)

where:

$I_{\rm tot}$	=	total daily incident solar radiation,
f	=	fraction of daylight (daylight hours/24),
t _d	=	time of day,
t _{sunrise}	=	time of sunrise.

Phytoplankton have been shown to be able to adapt to variations in light intensity (Steemann Nielsen et al., 1962; Steemann Nielsen and Park, 1964; Morel et al., 1987). Experimental data have indicated that phytoplankton take two to four days to adapt to changes in light intensity. Therefore, the value of I_s in Equation (2-9) is permitted to change as a function of the antecedent light history, according to the formula:

$$I_{s} = (I_{tot_{n-1}} + I_{tot_{n-1}} + I_{tot_{n-1}})/3$$
(2-13)

where:

$I_{tot_{n-3}}$	=	total solar radiation three days preceding current model day,
$I_{tot_{n-2}}$	=	total solar radiation two days preceding current model day,
$I_{tot_{n-1}}$	=	total solar radiation one day preceding current model day,

The nutrient saturated growth rate is then temperature-corrected using spatially dependent, values of ambient water column temperature. The temperature-corrected growth rate is computed using the following equation, which relates $\mu_{P_{max}}(T)$, the growth rate at ambient temperature, T, to $\mu_{P_{max}}(T_{opt})$, the growth rate at the optimal temperature, T_{opt} :

$$\mu_{\text{Pmax}}(T) = \mu_{\text{Pmax}}(T_{\text{opt}}) e^{-b_1 (T - T_{\text{opt}})^2} T \le T_{\text{opt}}$$
(2-14a)

or

$$\mu_{\text{Pmax}}(T) = \mu_{\text{Pmax}}(T_{\text{opt}}) e^{-b_1 (T - T_{\text{opt}})^2} T > T_{\text{opt}}$$
(2-14b)

and where β_1 is the effect of temperature below T_{opt} on growth and β_2 is the effect of temperature above T_{opt} on growth. A principal difference between the three phytoplankton groups is the magnitude of the T_{opt} . The nutrient saturated, temperature-corrected growth rate is then adjusted to reflect attenuation

due to nutrient levels. The effects of various nutrient concentrations on the growth of phytoplankton have been investigated, and the results are quite complex. As a first approximation to the effect of nutrient concentration on the growth rate, it is assumed that the phytoplankton population in question follows Monod growth kinetics with respect to the important nutrients. That is, at an adequate level of substrate concentration, the growth rate proceeds at the saturated rate for the ambient temperature and light conditions. However, at low substrate concentration, the growth rate becomes linearly proportional to substrate concentration. Thus, for a nutrient with concentration N_j in the jth segment, the factor by which the saturated growth rate is reduced in the jth segment is $N_j/(K_m + N_j)$. The constant, K_m , which is called the Michaelis, or half-saturation constant, is the nutrient concentration at which the growth rate is half the saturated growth rate. Since there are three nutrients, nitrogen, phosphorus and silica, considered in this framework, the Michaelis-Menten expression is evaluated for each nutrient and the minimum value is chosen to reduce the saturated growth rate,

$$G_{\mathbf{N}}(N) = Min\left(\frac{DIN}{K_{\mathbf{mN}} + DIN}, \frac{PO_{\mathbf{t}}}{K_{\mathbf{mP}} + PO_{\mathbf{t}}}, \frac{DSi}{K_{\mathbf{mSi}} + DSi}\right)$$
(2-15)

Three terms have been included in the modeling framework to account for the loss of phytoplankton biomass: endogenous respiration, sinking or settling from the water column and zooplankton grazing. Respiration has already been defined via Equation (2-3). The sinking of phytoplankton is an important contribution to the overall mortality of the phytoplankton population. Published values of the sinking velocity of phytoplankton, mostly in quiescent laboratory conditions, range from 0.1 to 18.0 m/day. In some instances, however, the settling velocity is zero or negative. Actual settling rates in natural waters are a complex phenomenon, affected by vertical turbulence, density gradients, and the physiological state of the different species of phytoplankton. An important factor shown to influence the physiological state of the algae is nutrient availability. Work by Bienfang et al. (1982) and Culver and Smith (1989) has shown that the settling rate of marine diatoms is increased primarily by low concentrations of silica, although low concentrations of nitrogen and low light availability were also found to increase diatom sinking rates. Although the net effective settling rate under nutrient stressed conditions is greatly reduced in relatively shallow, well-mixed regions of an estuary, sinking can contribute to the overall mortality of the algal population. In addition, the settling of phytoplankton can be a significant source of nutrients to the sediments and can play an important role in the generation of SOD. For these reasons, a term representing phytoplankton settling has been included in the algal mortality expression, and is determined by:

$$\mathbf{k}_{sp} = \frac{\mathbf{V}_{sPb}}{\mathbf{H}} + \frac{\mathbf{V}_{sPh}}{\mathbf{H}} (1 - \mathbf{G}_{N} (\mathbf{N}))$$
(2-16)

where k_{sp} is the net effective algal loss rate due to settling (day⁻¹), V_{sPb} is the base settling velocity of phytoplankton (m/day), V_{sPn} is the nutrient dependent settling rate (m/day), $G_N(N)$ is defined by Equation (2-15), and H is the depth of the model segment.

Zooplankton grazing may, depending upon the time of year and zooplankton biomass levels, be an important loss rate for phytoplankton. Rather than attempt to model the complex and dynamic processes of zooplankton grazing and growth, a simple first order loss rate representing the effect of zooplankton grazing on algal biomass is included in the model. The loss rate due to grazing is temperature corrected as per Equation (2-17),

$$k_{grz}(T) = k_{grz}(20^{\circ} C) \theta_{grz}^{(T-20)}$$
 (2-17)

where $k_{grz}(T)$ is the temperature corrected loss rate due to zooplankton grazing, k_{grz} (20°C) is the loss rate at 20°C and θ_{grz} is the temperature correction factor for zooplankton grazing. The units of k_{grz} are day⁻¹.

A principal component in the mass balance equation for the nutrient systems in the model eutrophication framework is the nutrient uptake associated with algal growth. In order to quantify the nutrient uptake it is necessary to specify the phytoplankton stoichiometry in units of nutrient uptake per mass of phytoplankton biomass synthesized. For carbon as the unit of phytoplankton biomass, the relevant ratios are the mass of nitrogen, phosphorus, and silica per unit mass of carbon. Lacking extensive measurements of the particulate forms of carbon, nitrogen, phosphorus and biogenic silica, this study assumed that the phytoplankton present in Massachusetts Bay are comprised of carbon and nutrients which approximate Redfield ratios; i.e., 106C:16N:1P (atomic), under nutrient saturated conditions. For silica, it was assumed that at nutrient saturated conditions the winter and fall diatoms had a carbon to silica ratio of 106C:18Si (atomic), while a ratio of 106C:6.5Si (atomic) was used for the summer functional group (recognizing that only a portion of the summer assemblage is comprised of diatoms).

However, while the use of Redfield ratios may be appropriate under nutrient saturated conditions, it has been shown (Antia et al., 1963; Caperon and Meyer, 1972; Chalup and Laws, 1990) that algae change their cellular composition or stoichiometry as a function of nutrient status. This is accounted for in the Laws/Chalup model via the following equations:

$$N_{x}: C = \left[QF + (1 - QF)(\mu / \mu_{Pmax})\right] / W_{Cx}$$

$$= 1 / W_{Cx} \text{ when } \mu = \mu_{Pmax}$$
(2-18)

and

$$Ch1:C = \frac{1 - (1 - QF)(1 - \mu / \mu_{Pmax}) - S/C - (\mu + k_{RB}/C)/[(1 - k_{RG})G_{prd}]}{W_{cChl}}$$

$$= \left\{ 1 - S/C - (\mu_{Pmax} + k_{RB}C) / [(1 - k_{RG})G_{prd}] \right\} / W_{cChl} \quad \text{when } \mu = \mu_{Pmax}$$
(2-19)

where:

N _x :C	=	the ratio of nutrient x (nitrogen, phosphorus or silica) to carbon,
QF	=	quotient of N_x :C values at relative growth rates of 0 and 1,
μ	=	the nutrient corrected growth rate ($\mu = \mu_{P_{max}} G_N(N)$),
W _{Cx}	=	the ratio of C to nutrient x in S, L, D,
Chl:C	=	the ratio of chlorophyll-a to C in L,
W_{CChl}	=	the ratio of C to chlorophyll-a in L.

The latter equation accounts for changes in the chlorophyll to carbon ratio both as a function of nutrient status and light. Equations (2-18) and (2-19) provide the equilibrium carbon to nutrient and carbon to chlorophyll ratios. However, as has been shown from experimental studies, there is a time period over which it takes the phytoplankton to reach new equilibrium conditions in response to changes in nutrient status and/or available light. This is accounted for in BEM by use of the following equations:

$$\frac{\mathrm{d}\mathbf{N}_{\mathbf{x}}:\mathbf{C}^{\mathbf{n}}}{\mathrm{d}t} = \mathbf{k}_{\mathbf{s}\mathbf{q}}(\mathbf{N}_{\mathbf{x}}:\mathbf{C}^{\mathbf{n}}_{\mathbf{s}\mathbf{q}} - \mathbf{N}_{\mathbf{x}}:\mathbf{C}^{\mathbf{n}})$$
(2-20)

$$\mathbf{N}_{\mathbf{x}}: \mathbf{C}^{\mathbf{n}+1} = \mathbf{N}_{\mathbf{x}}: \mathbf{C}^{\mathbf{n}} + dt \frac{d\mathbf{N}_{\mathbf{x}}: \mathbf{C}^{\mathbf{n}}}{dt}$$
(2-21)

and

$$\frac{\mathrm{dChl}:\mathrm{C}^{\mathbf{n}}}{\mathrm{dt}} = k_{sq}(\mathrm{Chl}:\mathrm{C}^{\mathbf{n}}_{sq} - \mathrm{Chl}:\mathrm{C}^{\mathbf{n}})$$
(2-22)

$$Chl: C^{n+l} = Chl: C^{n} + dt \frac{dChl: C^{n}}{dt}$$
(2-23)

where:

$N_x:C^n, N_x:C^{n+1}$	=	the nutrient to carbon ratios at time step n and $n + 1$, respectively
N _x :C ⁿ _{eq}	=	the equilibrium nutrient to carbon ratio at time step n, as
		determined from Equation (2-18)
k _{eq}	=	a constant which determines the time to achieve equilibrium,
Chl: C^n , Chl: C^{n+1}	=	the chlorophyll to carbon ratios at time stepn and $n + 1$,
		respectively
Chl:C ⁿ _{eq}	=	the equilibrium chlorophyll to carbon ratio at time step n, as
		determined from Equation (2-19).
dt	=	length of time step.

The model evaluates the nutrient to carbon and chlorophyll to carbon ratios to be used for the next time level based on the ratios at the current time level and the equilibrium ratios, determined from Equations (2-18) and (2-19), based upon environmental conditions at the current time level. A value of 1/day was chosen for k_{eq} based on the literature (Steemann Nielsen and Park, 1964; Antia et al., 1963; Caperon and Meyer, 1972). This corresponds to an equilibrium time of approximately 3 days.

Once the stoichiometric ratios have been determined, the mass balance equations may be written for the nutrients in much the same way as for the phytoplankton biomass. The principal processes determining the distribution of nutrients among the varius pools are: uptake of inorganic nutrients by phytoplankton for cell growth, the release of inorganic and organic nutrients algal respiration and predation processes, and the recycling of organic nutrients to inorganic forms via bacterial hydrolysis and mineralization.

Rather than attempt to model bacterial recycling or organic nutrients by including a bacterial system (for which there are little or no data against which to calibrate), a phytoplankton-dependent saturated recycle formulation was used. The assumption is made that bacterial biomass, and hence the

recycling rate, is proportional to the phytoplankton biomass. A number of field and laboratory studies (Hendry, 1977; Lowe, 1976, Menon et al., 1972; Jewell and McCarty, 1971) support this hypothesis. The saturated recycling relationship may be written

$$k(T) = k'(20^{\circ} C)\theta^{-(T-20)} \frac{P_{c}}{K_{mPc} + P_{c}}$$
(2-24)

where k(T) is the temperature corrected recycling rate, $k'(20^{\circ}C)$ is the saturated recycling rate at 20°C, P_c is the phytoplankton biomass, K_{mPc} is the half-saturation constant for recycling, θ is the temperature correction coefficient. Basically, this mechanism employs a quasi-first-order recycle that slows the recycling rate if the algal population is small, yet does not permit the recycling rate to increase in an unlimited fashion as phytoplankton biomass increases. Instead the mechanism permits zero-order recycling when the phytoplankton greatly exceed the half-saturation constant. The latter assumes that at higher population levels, other factors are limiting recycling rates or kinetics, so that it proceeds at its maximum zero-order rate.

2.2 COMPARISON OF ALGAL GROUPS

As presented in Section 2.1, the algal growth rate is dependent on the combination of temperature, light and nutrients. Other factors such as maximum growth, respiration rates, settling rates and zooplankton grazing will affect the algal biomass in the water column. The carbon to chlorophyll ratio can affect the amount of chlorophyll in the water column. In the original two-algal- group model, nine constants were used to differentiate between the two algal groups. These constants included the gross photosynthetic rate, the temperature optimum, the half-saturation constant for silica, the basal respiration rate, the base algal settling rate, the nutrient dependent algal settling rate, the nutrient saturated carbon to nitrogen ratio, the nutrient saturated carbon to silica ratio and the nutrient saturated carbon to chlorophyll ratio. With the addition of the third algal group, the original two phytoplankton groups' constants used for the model calibration were unchanged. The third algal group was given constants that would give it a competitive advantage over the other groups during the fall. The data from 1993 indicated that the fall bloom that occurred was composed primarily of the diatom Asterionellopsis glacialis, so the constants used for the winter diatoms were the starting point for the third algal group. Ultimately, six constants were modified for the fall diatom group compared to the winter group. These constants were the temperature optimum, the half-saturation constant for nitrogen, the half-saturation constant for silica, the base algal settling rate the nutrient saturated carbon to nitrogen ratio and the nutrient saturated carbon to chlorophyll ratio in L. Table 2-1 presents the constants that differed amongst the three algal groups. All the constants that were used for the algal kinetics are presented in Appendix A.

The six constants that were changed for the fall diatom group were modified to give this group the competitive advantage in the fall and not during other times of the year. Surface water temperatures during the period of the fall bloom were in the range of 10-15° C, so a temperature optimum of 14° C was assigned to this group. A temperature optimum at the higher end of the water temperature range was chosen to allow the fall diatom group to begin to gain a competitive advantage against the summer group, earlier in the fall. The half-saturation constant for nitrogen was reduced for the fall group to reduce the nutrient limitation growth reduction for the group. As nitrogen tends to be the limiting nutrient in Massachusetts and Cape Cod Bays during the fall, reducing the half-saturation constant for nitrogen allows the fall group to grow at a faster rate when nitrogen levels are low. The half-saturation constant for silica was increased for the fall diatom group. This was an attempt to limit the fall diatom group's growth during the spring when silica tends to be the limiting nutrient, but when the temperature is nearly ideal for the fall group. The base algal settling rate was decreased to reduce the loss of phytoplankton due to settling to the sediment and to depths where light is a limiting factor. Finally, the carbon to chlorophyll ratio was reduced based on data collected during the fall bloom in 1993.

It is important to note, however, that the carbon to chlorophyll ratios listed in Table 2-1 represent base values and are adjusted by the model, during the course of the simulation, depending upon ambient light and available nutrients. For example, the model computes mean carbon to chlorophyll ratios of 54, 83, and 20 mg C/mg Chl-a for the winter, summer, and fall functional groups versus estimates 76, 110, and 43 mg C/mg Chl-a for the same groups based on nearfield data collected in 1992 - 1994 and 1998 - 2001 (Libby, 2001). It is also worth noting that in the fall 1993 algal bloom, the carbon to chlorophyll ratio found at stations dominated by *Asterionellopsis glacialis* was about 20 mg C/mg Chl-a (Libby, 2001)

Additionally, boundary conditions had to be created for the fall diatom group. Without any data on which to base the boundary conditions, the fall diatom group was assigned at the boundary from the beginning of August to the end of October. Approximately the same amount of carbon associated with the fall diatom group was removed from the winter diatom and summer assemblage boundary conditions to keep a consistent concentration of phytoplankton carbon at the boundary. Fall diatom carbon concentrations assigned at the boundary are presented in Figure 2-1.

Tal	ble	2-1

Differences in Parameters Between the Three Algal Groups

Constant	Winter Diatom	Summer Assemblage	Fall Diatom	Units
Gross Photosynthetic Rate per Unit D	2.5	3.0	2.5	day -1
Temperature Optimum	8.	18.	14.	°C
Half-Saturation Constant for Nitrogen	0.010	0.010	0.005	mg N/L
Half-Saturation Constant for Silica	0.020	0.005	0.040	mg Si/L
Basal Respiration Rate	0.03	0.036	0.03	day ⁻¹
Base Algal Settling Rate	0.5	0.3	0.3	m/day
Nutrient Dependent Algal Settling Rate	1.0	0.7	1.0	m/day
Nutrient Saturated Carbon/Chlorophyll Ratio	40.	65.	15.	mg C/mg chl-a
Nutrient Saturated Carbon to Nitrogen Ratio	5.00	5.67	5.67	
Nutrient Saturated Carbon to Silica Ratio	2.5	7.0	2.5	



Figure 2-1. Fall Diatom Boundary Conditions

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SECTION 3

RESULTS

3.1 RESULTS OF PRELIMINARY CALIBRATION

A third algal group was added to the BEM kinetics and run for the years 1992 and 1993. 1993 was the year of primary concern due to an algal bloom that occurred during the fall. 1992 was modeled to provide initial conditions for 1993 and to show how the model reacts in a year without a fall bloom. The results of this exercise are presented below. These results must be considered preliminary because the resources were not available to attempt a complete calibration.

Figure 3-1 presents a comparison of model results to data for MWRA stations N16P, N17 and N21, in the middle of the near field area, in 1993. Overall, the model compares favorably with the data at this location. The model matches the surface chlorophyll quite well and does a better job of reproducing the fall phytoplankton bloom than the original calibration. However, the model still does not reproduce the chlorophyll-a maxima observed in mid-October. The revised model also better reproduces the measured POC values. The model compares well with the surface DO, but does not reproduce the high bottom DO concentrations observed during the summer. This problem was also evident in the original calibration. In general, the model reproduces the bottom water inorganic nutrient concentrations. The surface inorganic nutrients are reproduced by the model during the summer, but the model compares less favorably against the extreme low measurements observed during the spring and fall. However, the overall calibration at this location is quite good.

Figure 3-2 presents the model versus data comparison for the same location in 1992. From a phytoplankton standpoint, the model appears to do quite well reproducing both the surface chlorophyll and surface POC. However, the model slightly over-predicts chlorophyll concentrations observed in September, whereas the original model was more consistent with the data in September. Referring back to Figure 3-1, the revised model does compute slightly higher chlorophyll-a and POC concentrations in 1993 as compared to 1992. This may suggest some ability of the model to differentiate fall blooms between years. The data indicate fall peaks in chlorophyll-a in both 1992 and 1993, so the model compares reasonably for both years. For comparison the original calibration figures for stations N16P, N17 and N21 are presented in Appendix B.



Figure 3-1. 1993 Preliminary Calibration at HOM Stations N16P, N17 and N21 using Three Algal Group Kinetics

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3.2 COMPARISON TO PREVIOUS RESULTS

With the exception of the mid-October maxima, the revised model is able to reproduce the surface chlorophyll-a throughout the bays in 1993 as shown in Figure 3-3. The model compares favorably against the "high" year round chlorophyll-a concentrations in Boston Harbor at station F23P. The model also reproduces the spatial distribution in the near field by matching the data at stations N04P, N06, N07P and N10P. Spring and fall chlorophyll-a concentrations are reproduced in Cape Cod Bay at station F01P. The bloom is computed to extend into December in Cape Cod Bay, but there are no data to verify if the model is correct. The model does not match the high fall chlorophyll-a concentration at station F08, but the rest of the year is well calibrated.

The original calibration for chlorophyll-a in 1993 at the same six stations is presented in Figure 3-4 for comparison. It is obvious that in the initial calibration the model was not able to reproduce the fall chlorophyll-a concentrations, and that the model with the fall diatom group does a better job in this regard. However, even with the revised model framework, the model does not fully reproduce the maximum recorded chlorophyll-a data.

Figures 3-5 and 3-6 present model versus data comparisons for POC in 1993 for the new model and original calibration, respectively. Although there is a very noticeable difference between the results for chlorophyll-a, the model results for POC from the two runs are quite similar. The overall phytoplankton biomass does not differ significantly between the two model runs. This occurs because (1) the fall diatom group out-competes the summer and winter phytoplankton groups during the fall period, and since the fall group has a higher chlorophyll-a: carbon ratio, more chlorophyll is produced and (2) since algal biomass is nutrient- or yield-limited, approximately the same biomass (as carbon) is generated.

Aside from chlorophyll-a, the other constituent that changes noticeably between the two model runs is dissolved silica. This occurs because the fall diatom group requires more silica than does the summer assemblage. The results for the three algal group and two algal group models are presented in Figures 3-7 and 3-8, respectively. As these results are from a two year run, the silica concentrations at the beginning of 1993 are slightly lower in the three algal group model because of the carry over effects of the fall diatom bloom from the previous year. Again, 1993 silica concentrations at both the surface and bottom are lower in the fall in the three algal group model. These lower silica concentrations computed by the model improve the calibration to silica.



Figure 3-3. 1993 Preliminary Calibration Results for Chlorophyll-a at Six Locations within Massachusetts and Cape Cod Bays using Three Algal Group Kinetics







Figure 3-5. 1993 Preliminary Calibration Results for POC at Six Locations within Massachusetts and Cape Cod Bays

using Three Algal Group Kinetics







Figure 3-7. 1993 Preliminary Calibration Results for Dissolved Silica at Six Locations within Massachusetts and Cape Cod Bays

using Three Algal Group Kinetics





Other constituents, such as DIN, DIP and DO were practically unaffected by the addition of the third algal group. For this reason graphical comparisons between the original calibration and the preliminary third-algal-group model are not presented for these constituents.

3.3 SENSITIVITIES

In general, the addition of the third algal group improved the chlorophyll-a and silica calibration with little impact on the rest of the calibration. As discussed earlier, only a few parameters were changed to reproduce the fall bloom. The assigned boundary conditions were also used to reproduce the fall bloom. After several runs were completed to create the model calibration presented above, a few sensitivity runs were completed to discern the impact each particular parameter had on the results of the model. Results of these sensitivities for 1993 are presented in this section at three stations F23P in Boston Harbor, N16P in the near field area, and F01P in western Cape Cod Bay for both the surface and the bottom.

3.3.1 Carbon to Chlorophyll Ratio

The winter diatoms are assigned a carbon to chlorophyll ratio of 40. Since both the winter and fall phytoplankton groups are diatoms, the fall group was assigned a carbon to chlorophyll ratio of 40 in this sensitivity instead of 15 used in the new base calibration. Figure 3-9 shows that the chlorophyll-a concentrations are reduced during the fall in this scenario in Boston Harbor as would be expected. There is simply less chlorophyll-a associated with the phytoplankton carbon biomass. The POC concentrations are slightly higher in this scenario. As the total light extinction coefficient is a function of the background extinction and the self-shading of phytoplankton chlorophyll-a, and since there is less chlorophyll per unit carbon in the new sensitivity, there is less light limitation in this scenario, which allows the phytoplankton to grow to a higher level, producing more organic carbon. The dissolved oxygen concentrations are very similar between the two runs.

Figure 3-10 compares the results of this sensitivity with the base case for the inorganic nutrients in Boston Harbor. The silica, DIN and PO_4 concentrations are slightly lower during the fall in this scenario. The higher algal biomass consumes more inorganic nutrients in this scenario.

The comparison for chlorophyll, POC and DO in the near field area is presented in Figure 3-11. As in Boston Harbor, there is a noticeable difference in the fall chlorophyll levels. However, in the near field, there is not as much of a difference in the POC or DO concentrations. This is primarily due to the fact that the near field area is more nitrogen limited than light limited. Therefore, changes in the amount of available light, related to absorption by chlorophyll pigments, do not impact the growth of the phytoplankton very much. Inorganic nutrient concentrations do not differ much between the two runs (Figure 3-12).



Figure 3-9. Comparison of Computed Chl-a, POC and DO Concentrations between the Base Calibration and the C/Chl-a Sensitivity in Boston Harbor



Figure 3-10. Comparison of Computed Inorganic Nutrient Concentrations between the Base Calibration and the C/Chl-a Sensitivity in Boston Harbor



Figure 3-11. Comparison of Computed Chl-a, POC and DO Concentrations between the Base Calibration and the C/Chl-a Sensitivity in the Near Field Area


Figure 3-12. Comparison of Computed Inorganic Nutrient Concentrations between the Base Calibration and the C/Chl-a Sensitivity in the Near Field Area

The results in Cape Cod Bay for chlorophyll, POC and DO are presented in Figure 3-13. Chlorophyll-a concentrations are significantly lower in this sensitivity as compared to the base case. More POC is computed in this sensitivity especially in the bottom water. However, these changes have little impact on the dissolved oxygen. The small increase in the algal biomass reduces the inorganic nutrient concentrations in the fall as shown in Figure 3-14.

3.3.2 Temperature Optimum

The temperature optimum assigned to the fall diatom group was 14° C. Since the peak chlorophyll-a concentrations appeared to occur closer to 12° C, a sensitivity run was conducted with 12° C as the temperature optimum. The differences between the results of this sensitivity and the base case were minimal. For this reason the discussion of this sensitivity will be brief. Figure 3-15 presents the comparison between the two runs for chlorophyll, POC, and DO in western Cape Cod Bay where some of the largest differences were observed. Reducing the temperature optimum allowed the fall diatom group to persist longer at the end of 1992 and the beginning of 1993 as well as the end of 1993. Small increases in chlorophyll-a were computed during these periods as compared to the base run. The differences between the POC concentrations of these two runs are barely visible. Assigning a temperature optimum of either 12° C or 14° C seems equally valid.

3.3.3 Nitrogen Michaelis Constant

During the fall in many parts of Massachusetts and Cape Cod Bays nitrogen is the limiting nutrient for phytoplankton growth. A nitrogen Michaelis (or half-saturation growth) constant of 0.005 mg/L was assigned the fall diatom group to give this group a competitive advantage when inorganic nitrogen concentrations are low. For this sensitivity, a Michaelis constant of 0.01 mg/L was assigned to the fall diatom group, a value equal to that assigned to the other two phytoplankton groups.

Figure 3-16 presents the phytoplankton carbon concentration for each of the phytoplankton groups in Boston Harbor. In Boston Harbor, nitrogen is not as limiting as in other portions of the bays, therefore, the differences between the two runs are not significant. In the sensitivity run, the summer group has greater biomass than in the base case, and the fall diatom group has reduced biomass. The overall effect is only a small difference in the total phytoplankton biomass, but in the sensitivity run the summer group is better able to compete because it has a higher net growth rate when the nitrogen Michaelis constants are equal. Figure 3-17 shows that due to the shift in the phytoplankton biomass to the summer group, which has a lower chlorophyll-a : carbon ratio, the chlorophyll-a concentration decreases in the sensitivity. The POC and DO concentrations change only slightly with slightly more POC observed in the water column during the fall in the sensitivity. Figure 3-18 shows only small



Figure 3-13. Comparison of Computed Chl-a, POC and DO Concentrations between the Base Calibration and the C/Chl-a Sensitivity in Western Cape Cod Bay

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Figure 3-14. Comparison of Computed Inorganic Nutrient Concentrations between the Base Calibration and the C/Chl-a Sensitivity in Western Cape Cod Bay



Figure 3-15. Comparison of Computed Chl-a, POC and DO Concentrations between the Base Calibration and the Temperature Optimum Sensitivity in Western Cape Cod Bay







Figure 3-17. Comparison of Computed Chl-a, POC and DO Concentrations between the Base Calibration and the Nitrogen Michaelis Constant Sensitivity in Boston Harbor



Figure 3-18. Comparison of Computed Inorganic Nutrients Concentrations between the Base Calibration and the Nitrogen Michaelis Constant in Boston Harbor

differences in the inorganic nutrient concentrations between the two runs with the DIN and PO_4 concentrations declining and the DSi concentration increasing in the sensitivity.

In the near field area, as shown in Figure 3-19, the effect of changing the Michaelis constant is larger than in Boston Harbor, but still small. The fall diatom group does not develop as well in this sensitivity. Subsequently, the chlorophyll-a concentrations during the bloom are affected (Figure 3-20). However, the overall phytoplankton biomass changes very little so that the POC, DO, DIN and PO_4 are virtually unchanged (Figures 3-20 and 3-21). Only the DSi shows a difference as there is a shift from fall diatoms with a high silica requirement to the summer assemblage with a lower silica requirement.

The most dramatic effects of changing the Michaelis constant are seen in Cape Cod Bay. Figure 3-22 presents the phytoplankton carbon for each of the three groups. Here the effect is to cut the fall diatom biomass in half and double the summer assemblage biomass. Since the base carbon to chlorophyll ratios are so different between these two groups (65 vs. 15), there is a substantial difference between the fall chlorophyll concentrations between the two runs in Cape Cod Bay as shown in Figure 3-23. Again, the overall total phytoplankton biomass, as measured by phytoplankton carbon (Figure 3-22), does not change substantially because both groups require the same amount of nitrogen in their cells. The only difference is which group has the competitive advantage. Figure 3-24 shows that DIN and PO_4 change very little, but there is almost a 50 percent change in the DSi concentration in the fall at the surface.

3.3.4 Carbon to Nitrogen Ratio

A sensitivity run was conducted by changing the carbon to nitrogen ratio of the fall diatom group from 5.67 (the Redfield ratio) to 5.00 which is used for the winter diatom group. This change had virtually no impact on the run. Each phytoplankton group has the ability to vary its stoichiometric according to the constants specified in the model. Apparently, the modification of the base carbon to nitrogen ratio for the fall group did not change the relative competitive advantage for each phytoplankton group.

3.3.5 Maximum Saturated Growth Rate

The maximum saturated growth rate for the fall diatoms was modified from 2.5/day (same as winter diatoms) to 3.0/day (same as summer assemblage). The growth rate was modified to give the fall diatoms a greater competitive advantage for the limited resources in the fall. The results from the sensitivity were fairly uniform across the bays. Results from Boston Harbor will be presented as an example. From Figure 3-25, it can be observed that there is a small increase in the fall diatom carbon



Figure 3-19. Comparison of Computed Phytoplankton Carbon Concentrations between the Base Calibration and the Nitrogen Michaelis Constant Sensitivity in the Near Field Area



Figure 3-20. Comparison of Computed Chl-a, POC and DO Concentrations between the Base Calibration and the Nitrogen Michaelis Constant Sensitivity in the Near Field Area



Figure 3-21. Comparison of Computed Inorganic Nutrient Concentrations between the Base Calibration and the Nitrogen Michaelis Constant in the Near Field Area



Figure 3-22. Comparison of Computed Phytoplankton Carbon Concentrations between the Base Calibration and the Nitrogen Michaelis Constant Sensitivity in Western Cape Cod Bay



Figure 3-23. Comparison of Computed Chl-a, POC and DO Concentrations between the Base Calibration and the Nitrogen Michaelis Constant Sensitivity in Western Cape Cod Bay



Figure 3-24. Comparison of Computed Inorganic Nutrient Concentrations between the Base Calibration and the Nitrogen Michaelis Constant Sensitivity in Western Cape Cod Bay





at the expense of the summer assemblage. This translates into a small increase in the chlorophyll-a concentration in the fall as shown in Figure 3-26. However, the overall POC concentrations changes only minimally. With the slight increase in the fall diatom biomass, there is a small decrease in the DSi concentration in the fall as seen in Figure 3-27.

3.3.6 Settling Rate

The base settling rate for the fall diatoms was set at 0.3 m/day. For this sensitivity, the base algal settling rate for the fall diatoms was set to 0.5 m/day which is the same settling rate used for the winter diatoms. The overall effect of this change was small, so only Cape Cod Bay will be shown as an example. Not surprisingly, there is a decrease in the fall diatom biomass. The summer assemblage biomass increases slightly to fill the void in the fall (Figure 3-28). The chlorophyll-a concentration (Figure 3-29) declines slightly in the winter and fall. The POC concentration (Figure 3-29) declines slightly. There is no observable difference in the DO concentration. The DSi concentration, shown in Figure 3-30, increases slightly in the winter and fall. The DIN and PO₄ remain largely unaffected.

3.3.7 Silica Michaelis Constant

In the base run, the Michaelis constant for silica for the fall diatom group was set to 0.04 mg/L. This was an attempt to limit the possibility for the fall diatom group to grow during the spring period, when temperatures approach the temperature optimum of 14°C, but silica concentrations are low. For this sensitivity, the Michaelis constant was set to 0.02 mg/L. In Boston Harbor, silica tends to be the more potentially limiting nutrient, given the inputs of nitrogen from the Nut Island and Deer Island treatment facilities. Reducing the Michaelis constant for silica of the fall group, decreases the level of nutrient limitation for that group. The effect on the competition between the two diatom groups was negligible, as the biomass of the fall diatom group during the spring is small (Figure 3-31). During the fall, the fall diatom group increases its biomass at the expense of the summer assemblage. There is little change in the overall biomass, however. The shift in phytoplankton groups causes an increase in chlorophyll-a (Figure 3-32) and a decrease in the silica concentration (Figure 3-33). While silica does not tend to be as limiting outside of Boston Harbor, the results of this sensitivity were similar in the near field area and Cape Cod Bay.

3.3.8 Minimum Concentration at the Boundary

One of the more important factors controlling the biomass of the fall diatom group is the assigned boundary conditions. Fall diatom carbon was assigned at the boundary from the beginning of August to the end of October for the base case. During the rest of the year the boundary was



Figure 3-26. Comparison of Computed Chl-a, POC and DO Concentrations between the Base Calibration and the Maximum Saturated Growth Rate Sensitivity in Boston Harbor



Figure 3-27. Comparison of Computed Inorganic Nutrient Concentrations between the Base Calibration and the Maximum Saturated Growth Rate Sensitivity in Boston Harbor



Figure 3-28. Comparison of Computed Phytoplankton Carbon Concentrations between the Base Calibration and the Settling Rate Sensitivity in Western Cape Cod Bay



Figure 3-29. Comparison of Computed Chl-a, POC and DO Concentrations between the Base Calibration and the Settling Rate Sensitivity in Western Cape Cod Bay



Figure 3-30. Comparison of Computed Inorganic Nutrient Concentrations between the Base Calibration and the Settling Rate Sensitivity in Western Cape Cod Bay



Figure 3-31. Comparison of Computed Phytoplankton Carbon Concentrations between the Base Calibration and the Silica Michaelis Constant Sensitivity in Boston Harbor



Figure 3-3 . Comparison of Computed Chl-a P C and Con entrations between the Base Calibration and the Sili a Mi haelis Constant Sensitivity in Boston Harbor



Figure 3-33. Comparison of Computed Inorganic Nutrient Concentrations between the Base Calibration and the Silicas Michaelis Constant Sensitivity in Boston Harbor

assigned a concentration of zero. While running 1992 as a spin up allowed some biomass to remain at the beginning of 1993, there was no new biomass coming from the boundary. This differs from the other two phytoplankton groups which were assigned minimum concentrations of 0.005 mg/L at the top two standard levels and 0.002 mg/L in the bottom two standard levels at the boundary. For this sensitivity the fall diatom group was assigned the same minimum concentrations for the months outside of the August-October time period.

Figure 3-34 shows that in Boston Harbor the fall diatom biomass increases slowly into the fall with a larger increase occurring in the bottom waters. With the increase of the fall diatom biomass, both of the other phytoplankton groups have reduced biomass. The increase in fall diatom biomass, increases the chlorophyll-a concentration. However, Figures 3-35 and 3-36 show there was little change in the POC, DO and inorganic nutrient concentrations.

Figures 3-37, 3-38 and 3-39 present a similar story in the near field area. There are small increases in the fall diatom biomass at the surface with larger increases in biomass observed at depth. In Cape Cod Bay, there are significant increases in the fall diatom biomass in the bottom waters during July, August and September (Figure 3-40). This results in a large increase in the bottom water chlorophyll-a concentration (Figure 3-41), which is not consistent with the data. This change in biomass affects the summertime silica concentrations (Figure 3-42) and even the dissolved oxygen (Figure 3-41).

3.3.9 Equal Boundary Conditions

The boundary conditions that were assigned were based on assumptions that certain phytoplankton groups are advected into Massachusetts Bay during specific times of the year. Winter diatoms are assumed to be dominating the boundary waters in the spring, the summer assemblage in the summer and the fall diatoms would appear only in the late-summer through early fall. The boundary conditions can have a large influence on what happens in Massachusetts and Cape Cod Bays, so a final sensitivity was run where the phytoplankton carbon did not vary and was equally divided among the three groups. Each group was assigned a concentration of 0.05 mg/L, so the model constants would determine which group would dominate, not the boundary conditions.

Figure 3-43 presents the phytoplankton carbon for each of the groups in Boston Harbor. The effect of changing the boundary condition was to reduce the winter diatom biomass in the spring, allowing the summer group to increase its biomass earlier and to allow the fall diatom group to have a small presence throughout the year. The increased levels of the fall diatom group during the summer increase the light limitation and reduce the summer assemblage biomass in the bottom layer. In Figure 3-44 it can be observed that due to the increase in the fall diatom group during the year, the chlorophyll-



Figure 3-34. Comparison of Computed Phytoplankton Carbon Concentrations between the Base Calibration and the Minimum Boundary Condition Sensitivity in Boston Harbor



Figure 3-35. Comparison of Computed Chl-a, POC and DO Concentrations between the Base Calibration and the Minimum Boundary Condition Sensitivity in Boston Harbor



Figure 3-36. Comparison of Computed Inorganic Nutrient Concentrations between the Base Calibration and the Minimum Boundary Condition Sensitivity in Boston Harbor



Figure 3-37. Comparison of Computed Phytoplankton Carbon Concentrations between the Base Calibration and the Minimum Boundary Condition Sensitivity in the Near Field Area



Figure 3-38. Comparison of Computed Chl-a, POC and DO Concentrations between the Base Calibration and the Minimum Boundary Condition Sensitivity in the Near Field Area



Figure 3-39. Comparison of Computed Inorganic Nutrient Concentrations between the Base Calibration and the Minimum Boundary Condition Sensitivity in the Near Field Area



Figure 3-40. Comparison of Computed Phytoplankton Carbon Concentrations between the Base Calibration and the Minimum Boundary Condition Sensitivity in Western Cape Cod Bay



Figure 3-41. Comparison of Computed Chl-a, POC and DO Concentrations between the Base Calibration and the Minimum Boundary Condition Sensitivity in Western Cape Cod Bay



Figure 3-42. Comparison of Computed Inorganic Nutrient Concentrations between the Base Calibration and the Minimum Boundary Condition Sensitivity in Western Cape Cod Bay






Figure 3-44. Comparison of Computed Chl-a, POC and DO Concentrations between the Base Calibration and the Equal Boundary Condition Sensitivity in Boston Harbor

a concentrations are higher in the late spring and early summer. The changes in the boundary conditions have some effects on the inorganic nutrient concentrations (Figure 3-45), but the changes are not large. The results in the near field area are similar to those in Boston Harbor.

Figures 3-46 through 3-48 present the results in Cape Cod Bay. The biomass of the winter diatoms is largely unaffected in the surface layer. In the bottom layer, the winter diatom biomass is cut in half starting in May. The summer assemblage biomass is greatly reduced during the summer. The fall diatom group does extremely well at both the surface and the bottom and dominates the algal biomass during the second half of the year. The increase in the fall diatom biomass causes high (for Cape Cod Bay) concentrations of chlorophyll-a in both the surface and bottom layers of the water column during the summer, which is inconsistent with the observed data. The changes in POC and DO are minor. Dissolved silica is impacted during the summer and early fall, but the DIN and PO_4 are largely unaffected.



Figure 3-45. Comparison of Computed Inorganic Nutrient Concentrations between the Base Calibration and the Equal Boundary Condition Sensitivity in Boston Harbor



Figure 3-46. Comparison of Computed Phytoplankton Carbon Concentrations between the Base Calibration and the Equal Boundary Condition Sensitivity in Western Cape Cod Bay



Figure 3-47. Comparison of Computed Chl-a, POC and DO Concentrations between the Base Calibration and the Equal Boundary Condition Sensitivity in Western Cape Cod Bay

livity in western cape C



Figure 3-48. Comparison of Computed Inorganic Nutrient Concentrations between the Base Calibration and the Equal Boundary Condition Sensitivity in Western Cape Cod Bay

SECTION 4

CONCLUSIONS

The fall bloom analysis described in this report focused primarily on a species with a very low carbon to chlorophyll ratio. The data collected in the fall of 1993 pointed to the conclusion that an algal species such as this was the cause for the high chlorophyll-a levels observed during this period. While the chlorophyll-a levels were high, the POC concentrations did not differ significantly from other fall periods in other years. However, a species with a low carbon to chlorophyll ratio is not the only possible cause of a fall algal bloom.

Fall blooms can occur if there is an increase in the available nutrients either via influx from the boundary, an upwelling event or an early fall turnover of the water column. These particular events did not appear to occur in 1993, but it would be important for the model to be able to reproduce these events if they did occur. Another apparent cause of a fall bloom would be a large influx of algae from the Gulf of Maine. While this may appear as if there was a great deal of biological productivity in the bays, the growth would have actually occurred outside the bays. It is possible that physical conditions play a role in fall phytoplankton blooms. Perhaps particularly quiescent or still conditions favor one species over another. This is something that is not directly incorporated in the model other than the transport of phytoplankton towards or away from their preferred light, temperature, and nutrient conditions. These factors make modeling specific algal species quite difficult.

This modeling exercise has shown that the addition of a third algal group to the model can enable the model to reproduce some but not all of the features of the 1993 fall bloom. Comparisons to the original 1993 calibration show that the model calibration to chlorophyll-a and dissolved silica can be improved without adversely affecting the calibration to the other constituents. While the several model constants that were modified helped the fall diatoms succeed during the fall, the two inputs that seem to have the greatest impact are the carbon to chlorophyll ratio and the boundary conditions. This being the case, the question becomes, "Does the addition of the third algal group improve the model's predictive capability?", and; "Does the model reproduce the fall bloom for the right reasons?" Running the year 1992 with the same model constants and fall diatom boundary conditions as were used for 1993 resulted in similar results as the 1993 run in terms of chlorophyll-a concentrations. While the third algal group improves the calculation of a fall bloom, it is likely the model would compute a fall bloom every year.

As was noted above, the addition of the third algal group improved the calibration for the fall

of 1993, but still did not fully reproduce the maximum concentrations of phytoplankton chlorophyll observed in the data in mid-October. The addition of the third algal group produced mixed results when comparing model computations against observed data in 1992 (Figure 3-2 vs. Figure B-1). It appeared that the revised model provided a slightly better calibration against the October 1992 chlorophyll data, but adversely affected the calibration in September 1992, when the model overestimated the observed chlorophyll data. It is interesting to note, however, that the two functional algal group version of BEM and the three functional algal group version of the model compute approximately the same algal biomass in carbon units, and it is important to remember that it is carbon, not chlorophyll, which determines oxygen production and respiration. So the answers to the posed questions are mixed. Overall, it can be argued that the addition of the fall algal group does improve the 1993 calibration to the observed fall data (but still does not fully reproduce the observed maxima) without causing significant adverse effects on the calibration to the 1992 data set. However, it is important to note that the 1993 fall algal bloom was largely dominated by Asterionellopsis glacialis and that the revised BEM, i.e., the addition of the third algal group, does not purport to model Asterionellopsis. This cannot be done, until marine biologists can develop a full understanding of the physical, chemical, and biological dynamics that permit Asterionellopsis to dominate the Gulf of Maine/Massachusetts Bays phytoplankton community in some years and not in others.

Further research and/or calibration should be considered before this modeling framework can be used for projection purposes. In this analysis, only the constants and parameters for the third algal group were adjusted. It is possible that the addition of the third algal group may require adjustment, in other parameters and constants previously calibrated in the two algal group model. Also, the model should be tested/calibrated for a year that clearly had no fall algal bloom, so the model can be refined to the point so that it would compute fall algal blooms only in years that they occurred. The results from incorporating a third algal group are promising. However, further research should be conducted.

SECTION 5

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APPENDIX A PHYTOPLANKTON NET GROWTH EQUATIONS

Net Growth Rate

$$\mathbf{G}_{\text{net}} = (\mu_{\text{Pmax}}(\mathbf{T}) \cdot \mathbf{G}_{\text{N}}(\mathbf{N}) - \mathbf{k}_{\text{RB}} - \mathbf{k}_{\text{sP}}(\mathbf{T}) - \mathbf{k}_{\text{grz}}(\mathbf{T})) \cdot \mathbf{P}_{\text{c}}$$

Specific Growth Rate

$$G_{p} = \mu_{Pmax} (T) \cdot G_{N} (N)$$

Nutrient Saturated Growth Rate

$$----\mu_{\text{Pm ax}} = \frac{G_{\text{pre}} \cdot (1-k_{\text{RG}}) \cdot (1-S/C) \cdot I}{G_{\text{pre}} / G_{\text{prlo}} + I(1+G_{\text{pre}} / I_s G_{\text{prlo}})} - k_{\text{RB}}$$

Temperature Correction

$$\begin{split} \mu_{\text{Pmax}} \left(T \right) &= \mu_{\text{Pmax}} \left(T_{\text{opt}} \right) \cdot e^{-\beta_{\text{f}} \cdot \left(T - T_{\text{opt}} \right)^2} & T \leq T_{\text{opt}} \\ \mu_{\text{Pmax}} \left(T \right) &= \mu_{\text{Pmax}} \left(T_{\text{opt}} \right) \cdot e^{-\beta_{\text{f}} \cdot \left(T - T_{\text{opt}} \right)^2} & T > T_{\text{opt}} \end{split}$$

Light Attenuation

$$I(z,t) = I_{suff}(t) e^{-k_s z}$$

<u>Average Light</u>

$$I(z, t) = \frac{I_{sumf}(t)}{k_e H} (1 - e^{-k_e z})$$
$$k_e = k_{e_{buse}} + k_c \cdot a_{ChlC} \cdot P_c$$

$$I_{sunf(t)} = \frac{I_{tot}}{0.635 \cdot f} sin\left(\frac{p(t_d - t_{sunrise})}{f}\right)$$
$$I_s = \left(I_{tot_{n-1}} + I_{tot_{n-1}} + I_{tot_{n-1}}\right)/3$$

Chlorophyll to Carbon Ratio (a_{ChlC})

$$a_{\rm ChiC} = \frac{1 - (1 - QF)(1 - \mu / \mu_{\rm Pmax}) - S / C - (\mu + k_{\rm RB} / C)(1 - k_{\rm RG})G_{\rm pre}}{W_{\rm CChi}} - \frac{1 - (1 - QF)(1 - \mu / \mu_{\rm Pmax}) - S / C - (\mu + k_{\rm RB} / C)(1 - k_{\rm RG})G_{\rm pre}}{W_{\rm CChi}} - \frac{1 - (1 - QF)(1 - \mu / \mu_{\rm Pmax}) - S / C - (\mu + k_{\rm RB} / C)(1 - k_{\rm RG})G_{\rm pre}}{W_{\rm CChi}} - \frac{1 - (1 - QF)(1 - \mu / \mu_{\rm Pmax}) - S / C - (\mu + k_{\rm RB} / C)(1 - k_{\rm RG})G_{\rm pre}}{W_{\rm CChi}} - \frac{1 - (1 - QF)(1 - \mu / \mu_{\rm Pmax}) - S / C - (\mu + k_{\rm RB} / C)(1 - k_{\rm RG})G_{\rm pre}}{W_{\rm CChi}} - \frac{1 - (1 - QF)(1 - \mu / \mu_{\rm Pmax}) - S / C - (\mu + k_{\rm RB} / C)(1 - k_{\rm RG})G_{\rm pre}}{W_{\rm CChi}} - \frac{1 - (1 - QF)(1 - \mu / \mu_{\rm Pmax}) - S / C - (\mu + k_{\rm RB} / C)(1 - k_{\rm RG})G_{\rm Pre}}{W_{\rm CChi}} - \frac{1 - (1 - QF)(1 - \mu / \mu_{\rm Pmax}) - S / C - (\mu + k_{\rm RB} / C)(1 - k_{\rm RG})G_{\rm Pr}}{W_{\rm CChi}} - \frac{1 - (1 - QF)(1 - \mu / \mu_{\rm RG})}{W_{\rm CChi}} - \frac{1 - (1 - QF)(1$$

Nutrient Uptake

$$G_{N}(N) = Min\left(\frac{DIN}{K_{mN} + DIN}, \frac{DIP}{K_{mP} + DIP}, \frac{Si}{K_{mSi} + Si}\right)$$

DIN	=	dissolved inorganic nitrogen = $NH_3 + NO_2 + NO_3$
DIP	=	dissolved inorganic phosphorus
Si	=	available silica

Endogenous Respiration

$$k_{PR} = \frac{k_{RB} + k_{RG} + \mu}{1 - k_{RG}}$$

$$\mu = G_N (N) \cdot \mu_{Pmax}$$

Algal Settling

$$\mathbf{k}_{\mathfrak{sP}}(\mathbf{T}) = \left(\frac{\mathbf{v}_{\mathfrak{sPb}}}{\mathbf{H}} + \frac{\mathbf{v}_{\mathfrak{sPh}}}{\mathbf{H}} \cdot (1 - \mathbf{G}_{\mathbf{N}}(\mathbf{N}))\right) \cdot \boldsymbol{\theta}_{\mathfrak{sP}}^{(\mathbf{T}-20)} - \mathbf{h}_{\mathbf{N}}^{(\mathbf{T}-20)} - \mathbf{h}_{\mathbf{N}}^{(\mathbf{T}$$

Zooplankton Grazing

 $k_{gra}(T) = k_{gra}(20^{\circ} \text{ C}) \cdot \theta_{gra}^{(T-20)}$

<u>Exogenous Variables</u>								
Description	<u>Notation</u>	<u>Units</u>						
Total Extinction Coefficient	k _e	m^{-1}						
Base Extinction Coefficient	k _{ebase}	m^{-1}						
Total Daily Surface Solar Radiation	I_{tot}	langleys/day						
Temperature	Т	°C						
Segment Depth	Н	m						
Fraction of Daylight	f	day						
Time of Day	t_{d}	day						
Time of Sunrise	t _{sunrise}	day						
Structural Carbon	S	g/cell						
Reservoir Carbon	R	g/cell						
Carbon Associated with the Light Reactions of Photosynthesis	L	g/cell						
Carbon Associated with the Dark Reactions of Photosynthesis	D	g/cell						
Total Cell Carbon	С	g/cell						
Irradiance	Ι	mol quanta/m²-d						
Value of I when $G_{prls} G_{prlo/2}$	I_s	mol quanta/m²-d						
Growth Rate	μ	day ⁻¹						
Value of G _{prl} Under Nutrient-Saturated Conditions	G_{prls}	m²/mol quanta						
Gross Rate of Photosynthesis Per Unit L Per Unit Light Intensity	G_{prl}	m²/mol quanta						
Phytoplankton Biomass	P_{c}	mg C/L						
Total Algal Respiration Rate	k_{PR}	day ⁻¹						
Net Growth Rate	G_{net}	day ⁻¹						
Specific Growth Rate	G_{p}	mg C /day						

Description	Notation	Winter Diatoms	Summer Assemblage	Fall Diatoms	Units
Gross photosynthetic rate per unit D	μ _{bre}	2.5	3.0	2.5	day ⁻¹
Gross photosynthetic rate per unit L per unit light intensity in the limit of zero irradiance	G _{prd}	0.28	0.28	0.28	L _m /mol quanta
Quotient of nutrient to carbon ratios at relative growth rates of 0 and 1	QF	0.85	0.85	0.85	
Effect of Temperature below $\rm T_{opt}$ on growth	β_1	.004	.004	.004	°C ⁻²
Effect of Temperature above T_{opt}	β_2	.006	.006	.006	°C ⁻²
Temperature Optimum	$\mathrm{T}_{\mathrm{opt}}$	8.	18.	14.	°C
Phytoplankton Self-Shading Atttenuation	k _c	0.017	0.017	0.017	m ² /mg chl-a
Half-Saturation Constant for Nitrogen	K_{mN}	0.010	0.010	0.005	mg N/L
Half-Saturation Constant for Phosphorus	K _{mP}	0.001	0.001	0.001	mg P/L
Half-Saturation Constant for Silica	K _{mSi}	0.020	0.005	0.040	mg Si/L
Growth Related Respiration Coefficient	k _{RG}	0.28	0.28	0.28	
Basal Respiration Rate	k _{RB}	0.03	0.036	0.03	day ⁻¹
Base Algal Settling Rate	v _{sPb}	0.5	0.3	0.3	m/day
Nutrient Dependent Algal Settling Rate	v _{sPn}	1.0	0.7	1.0	m/day
Temperature Coefficient	θ_{sP}	1.027	1.027	1.027	(at 20 °C)
Loss Due to Zooplankton Grazing	k _{grz}	0.1	0.1	0.1	day ⁻¹
Temperature Coefficient	$\theta_{ m grz}$	1.10	1.10	1.10	(at 20 °C)
Carbon/Chlorophyll Ratio in L	W _{CChl}	40.	65.	15.	mg C/mg chl-a
Nutrient Saturated Carbon to Phosphorus Ratio	W_{CP}	40.	40.	40.	
Nutrient Saturated Carbon to Nitrogen Ratio	W_{CN}	5.0	5.67	5.67	
Nutrient Saturated Carbon to Silica Ratio	W _{CS}	2.5	7.0	2.5	
Ratio of Structural Carbon to Total Cell Carbon	S/C	0.1	0.1	0.1	

Appendix A. Phytoplankton Net Growth Equations

APPENDIX B ORIGINAL CALIBRATION FIGURES



